NAG Toolbox for MATLAB

f08cj

1 Purpose

f08cj generates all or part of the real n by n orthogonal matrix Q from an RQ factorization computed by f08ch.

2 Syntax

```
[a, info] = f08cj(a, tau, 'm', m, 'n', n, 'k', k)
```

3 Description

f08cj is intended to be used following a call to f08ch, which performs an RQ factorization of a real matrix A and represents the orthogonal matrix Q as a product of k elementary reflectors of order n.

This function may be used to generate Q explicitly as a square matrix, or to form only its trailing rows.

Usually Q is determined from the RQ factorization of a p by n matrix A with $p \leq n$. The whole of Q may be computed by:

```
[a, info] = f08cj(a, tau);
```

(note that the array **a** must have at least n rows), or its trailing p rows as:

```
[a, info] = f08cj(a(1:p,:), tau, 'k', p);
```

The rows of Q returned by the last call form an orthonormal basis for the space spanned by the rows of A; thus f08ch followed by f08cj can be used to orthogonalise the rows of A.

The information returned by f08ch also yields the RQ factorization of the trailing k rows of A, where $k < p$. The orthogonal matrix arising from this factorization can be computed by:

```
[a, info] = f08cj(a, tau, 'k', k);
```

or its leading k columns by:

```
[a, info] = f08cj(a(1:k,:), tau, 'k', k);
```

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,$*$) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the vectors which define the elementary reflectors, as returned by f08ch.

2: **tau**$(*)$ **– double array**

Note: the dimension of the array **tau** must be at least $\max(1, \mathbf{k})$.

tau(i) must contain the scalar factor of the elementary reflector H_i, as returned by f08ch.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix Q.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix Q.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

3: **k – int32 scalar**

Default: The dimension of the array **tau**.

k, the number of elementary reflectors whose product defines the matrix Q.

Constraint: $\mathbf{m} \geq \mathbf{k} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a**(**lda,***$*$*) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix Q.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows: 1: **m**, 2: **n**, 3: **k**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O\epsilon$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating point operations is approximately $4mnk - 2(m+n)k^2 + \frac{4}{3}k^3$; when $m = k$ this becomes $\frac{2}{3}m^2(3n-m)$.

The complex analogue of this function is f08cw.

9 Example

```
a  =  [-0.2536561223671959,  -0.09569528025093453,  -2.894845895288455,  -
0.5041099118295458, ...
     1.921275206091976, -0.8729501828758776;
         -0.7751853859846198,  0.2275943648113135,  0.2084279305583289,  -
1.581266529610948, ...
     -1.053208461749375, 0.9018026937371963;
           0.1094532397645297,  0.5121810603924651,  -0.0952450839256265,
0.2476000680518517, 1.692758571755385, 0.213935545781127;
         0.06960714430288747,  -0.5958371552327169,  -0.09745000202404246,
0.02227428617692399, ...
     -0.5930528694606012, -3.091585353827385];
tau = [1.863066450840819;
     1.179138368141079;
     1.487335114645626;
     1.16172932097152];
[aOut, info] = f08cj(a, tau)
```

```
aOut =
   -0.0833    0.2972   -0.6404    0.4461   -0.2938   -0.4575
    0.9100   -0.1080   -0.2351   -0.1620    0.2022   -0.1946
   -0.2202   -0.2706    0.2220   -0.3866    0.0015   -0.8243
   -0.0809    0.6922    0.1132   -0.0259    0.6890   -0.1617
info =
           0
```